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MASS TRANSFER IN TURBULENT PIPE FLOW OF VISCOELASTIC FLUIDS

Y. I. CHO and J. P. HARTNETT

Energy Resources Center, University of Illinois at Chicago Circle, Box 4348, Chicago, Il 60680, U.S.A.

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Abstract—The mass transfer entry length and maximum mass transfer reduction asymptote for drag-reducing viscoelastic fluids have been calculated analytically using the successive approximation technique. The eddy diffusivity of mass reported by Shulman *et al.* [10] and Virk's velocity profile corresponding to the maximum mass transfer reduction in turbulent pipe flow were applied to the present analysis. The mass transfer entry length for these fluids was found to be 8–40 pipe diameters depending on the Reynolds number. The predicted mass transfer rates for these fluids are in good agreement with available empirical mass transfer results, which show approximately 65–75% reduction in the mass transfer rate compared to the Newtonian values at the same Reynolds and Schmidt numbers. A comparison of these mass transfer results with available heat transfer measurements leads to the conclusion that there is no simple direct relationship between heat and mass transfer for drag-reducing viscoelastic fluids.

NOMENCLATURE

- $\bar{A}$, dimensionless value defined by equation (17);
- d, diameter of tube;
- D, diffusion coefficient;
- f, Fanning friction coefficient;
- K, mass transfer coefficient;
- j_D, the mass transfer j factor, $(Sh/ReSc)\cdot Sc^{2/3}$;
- j_H, the heat transfer j factor, $(Nu/RePr)\cdot Pr^{2/3}$;
- l, total tube length;
- L, entrance length;
- $\dot{m}$, uniform mass flux at the wall;
- r, radial coordinate;
- R, radius of tube;
- Re, Reynolds number defined by equation (14);
- Sc, Schmidt number, ν/D;
- Sh, Sherwood number, Kd/D;
- u, axial velocity;
- u^*, friction velocity, $\sqrt{\tau_w/\rho}$;
- w, mass fraction;
- w_{in}, inlet mass fraction;
- x, axial coordinate;
- y, radial distance from the tube wall, $R - r$.

Greek symbols

- Δ, diffusional boundary layer thickness;
- $\varepsilon_D, \varepsilon_H, \varepsilon_M$, eddy diffusivity of mass, heat and momentum respectively;
- ν, kinetic viscosity evaluated at the tube wall;
- ρ, mass density of working fluid;
- τ_w, wall shear stress.

Subscripts

- b, bulk parameter;
- D, designates mass transfer;
- H, designates heat transfer;
- M, designates momentum transfer;
- x, local value along the axis;
- 1, the first approximation;
- 2, the second approximation.

Superscripts

- $+$, dimensionless variables defined by equation (6);
- $'$, differentiation with respect to x^+.

INTRODUCTION

A NUMBER of different analytical techniques have been used to solve the mass transfer problem in the entry region of laminar channel flows. As pointed out by Popel and Gross [1] these analytical approaches can be classified generally into two categories

(1) the extension of the Graetz problem by computing a large number of eigenvalues and eigenfunctions;
(2) the extension of Leveque's similarity solution in a power series asymptotic expansion.

Notter and Sleicher [2] applied the extension of the Graetz problem to fully developed turbulent pipe flows for Newtonian fluids with the boundary condition of constant wall concentration and derived the following correlation by calculating the corresponding eigenvalues and eigenfunctions numerically

$$j_D = 0.149Re^{-0.12} \quad Re > 5000 \text{ and } Sc > 100. \quad (1)$$

Dimant and Poreh [3] also used the extension of the Graetz problem to predict the local turbulent heat transfer rates of drag-reducing viscoelastic fluids for both constant wall temperature and constant heat flux boundary conditions under the assumption of $\varepsilon_M = \varepsilon_H$.

In the current study, we develop a procedure which makes it possible to obtain an analytical prediction of the mass transfer entry length and the fully developed mass transfer coefficient for undegraded saturated viscoelastic fluids in turbulent pipe flow. Successive approximation [4] is applied for the case of constant mass flux boundary condition (the so-called boundary condition of the second kind).

ANALYSIS

Consider a viscoelastic fluid in turbulent flow through a circular tube with mass transfer taking place between the fluid and the tube wall. Under the assumptions of steady flow, axisymmetric diffusion, fully developed hydrodynamic condition, no chemical reaction and uniform mass flux at the wall ($\dot{m}$), the governing mass diffusion equation and corresponding boundary conditions in the developing region can be written as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r(D+\varepsilon_D)\frac{\partial w}{\partial r}\right]=u\frac{\partial w}{\partial x} \tag{2}$$

$$w = w_{\text{in}} \quad \text{at } x = 0 \tag{3}$$

$$w = w_{\text{in}} \quad \text{at } x > 0, \quad r = R - \Delta \tag{4}$$

$$\rho D\frac{\partial w}{\partial r} = \dot{m} \quad \text{at } x > 0, \quad r = R \tag{5}$$

where $\dot{m}$ is the constant mass flux at the wall. Let us now introduce the following dimensionless variables

$$w^+ = \frac{\rho(w - w_{\text{in}})u^*}{\dot{m}}, \quad x^+ = \frac{xu^*}{\nu},$$

$$y^+ = \frac{yu^*}{\nu}, \quad u^+ = \frac{u}{u^*} \tag{6}$$

where $y = R - r$ and u^* is the friction velocity defined as $\sqrt{\tau_w/\rho}$. Then, the mass diffusion equation and the three boundary conditions can be non-dimensionalized as follows:

$$\frac{1}{R^+ - y^+}\frac{\partial}{\partial y^+}\left[(R^+ - y^+)\left(\frac{1}{Sc}+\frac{\varepsilon_D}{\nu}\right)\frac{\partial w^+}{\partial y^+}\right] = u^+\frac{\partial w^+}{\partial x^+} \tag{7}$$

$$w^+ = 0 \quad \text{at } x^+ = 0 \tag{8}$$

$$w^+ = 0 \quad \text{at } x^+ > 0, \quad y^+ = \Delta^+ \tag{9}$$

$$\frac{1}{Sc}\frac{\partial w^+}{\partial y^+} = -1 \quad \text{at } x^+ > 0, \quad y^+ = 0 \tag{10}$$

where $Sc = \nu/D$ and Δ^+ is the diffusional boundary layer thickness.

The successive approximation technique used in the present study requires explicit expressions for the velocity profile (u^+) and the eddy diffusivity of mass (ε_D). It is assumed that the fluid is undegraded and saturated and the velocity profile and eddy diffusivity of mass correspond to this condition. Then, the resulting prediction will yield the maximum mass transfer reduction asymptote.

Virk's three layer velocity model† [5, 6] for maximum drag reduction corresponds to the stated condition and will be applied in the right hand side of equation (7)

$$\left.\begin{aligned} u^+ &= y^+ && y^+ \le 11.6 \\ u^+ &= 11.7 \ln y^+ - 17.0 && 11.6 \le y^+ \le y_n^+ \\ u^+ &= 2.5 \ln y^+ + 9.2 \ln y_n^+ - 17.0 && y_n^+ \le y^+ \end{aligned}\right\} \tag{11}$$

where y_n^+ is the distance to the interface between the buffer zone and the turbulent core. For the maximum drag reduction case y_n^+ is equal to R^+.

Shulman *et al.* [9] reported the eddy diffusivity of mass (ε_D) as a function of dimensionless radial distance y^+ for saturated and undegraded polymer solutions. Recently, Shulman and Pokryvailo [10] modified their original expression and proposed the following:

$$\frac{\varepsilon_D}{\nu} = 1.6 \times 10^{-4} y^{+3.0}. \tag{12}$$

Equation (12) will be used for the eddy diffusivity of mass in the present analysis.

For fully established mass transfer conditions corresponding to the imposed boundary condition of the second kind it can be shown that

$$\frac{\partial w^+}{\partial x^+} = \frac{4}{Re} \tag{13}$$

where

$$Re = \frac{u_b d}{\nu} \tag{14}$$

and ν is the kinetic viscosity evaluated at the tube wall. Substituting equation (13) into (7), integrating w^+ twice with respect to y^+ and applying the boundary conditions, the first approximation of the mass fraction profile becomes

$$w_1^+ = \int_{y^+}^{\Delta^+}\left[R^+\Bigg/(R^+ - y^+)\left(\frac{1}{Sc}+\frac{\varepsilon_D}{\nu}\right)\right]\mathrm{d}y^+ - \int_{y^+}^{\Delta^+}\left\{\left[\int_0^{y^+}\frac{4}{Re}u^+(R^+ - \Delta^+)\,\mathrm{d}y^+\right]\Bigg/ (R^+ - y^+)\left(\frac{1}{Sc}+\frac{\varepsilon_D}{\nu}\right)\right\}\mathrm{d}y^+. \tag{15}$$

A second approximation can be derived by substituting w_1^+ into the right hand side of equation (7) and repeating the above-mentioned procedure.

$$w_2^+ = \int_{y^+}^{\Delta^+}\left[R^+\Bigg/(R^+ - y^+)\left(\frac{1}{Sc}+\frac{\varepsilon_D}{\nu}\right)\right]\mathrm{d}y^+$$

† It is worth mentioning that the velocity profiles proposed by Yoo [7] and Ng [8] for maximum drag reduction were also applied in the current study and the final results were found to be almost identical to those obtained using Virk's profile.

$$-\bar{A}\int_{y^+}^{\Delta^+}\left\{\left[\int_0^{y^+} u^+(R^+-\Delta^+)\mathrm{d}y^+\right]\Big/ (R^+-y^+)\left(\frac{1}{Sc}+\frac{\varepsilon_D}{\nu}\right)\right\}\mathrm{d}y^+ \quad (16)$$

where

$$\bar{A}=\left[\Delta^{+\prime}\Big/(R^+-\Delta^+)\left(\frac{1}{Sc}+\frac{\varepsilon_D}{\nu}\right)\right] \times\left[R^+-\int_0^{\Delta^+}\frac{4}{Re}u^+(R^+-y^+)\mathrm{d}y^+\right]. \quad (17)$$

Thus, the mass fraction distribution along the wall can be obtained by replacing the lower limit y^+ in equation (16) with zero.

$$w_w^+=\int_0^{\Delta^+}\left[R^+\Big/(R^+-y^+)\left(\frac{1}{Sc}+\frac{\varepsilon_D}{\nu}\right)\right]\mathrm{d}y^+ -\bar{A}\int_0^{\Delta^+}\left\{\left[\int_0^{y^+}u^+(R^+-\Delta^+)\mathrm{d}y^+\right]\Big/ (R^+-y^+)\left(\frac{1}{Sc}+\frac{\varepsilon_D}{\nu}\right)\right\}\mathrm{d}y^+. \quad (18)$$

$\bar{A}$ can be simplified using $\partial w_2^+/\partial y^+|_{y^+=\Delta^+}=0$, which renders a useful form for $\Delta^{+\prime}$.

$$\Delta^{+\prime}=\frac{\mathrm{d}\Delta^+}{\mathrm{d}x^+}=R^+(R^+-\Delta^+)\left(\frac{1}{Sc}+\frac{\varepsilon_D}{\nu}\right)\Big/ \left\{\left[\int_0^{\Delta^+}u^+(R^+-y^+)\mathrm{d}y^+\right] \times\left[R^+-\int_0^{\Delta^+}u^+(R^+-y^+)\mathrm{d}y^+\right]\right\}. \quad (19)$$

Therefore, $\bar{A}$ becomes

$$\bar{A}=R^+\Big/\int_0^{\Delta^+}u^+(R^+-y^+)\mathrm{d}y^+. \quad (20)$$

From equation (19), the dimensionless mass transfer entry length L^+ for drag-reducing viscoelastic fluids can be derived as follows:

$$L^+=\int_0^{R^+}\left\{\left[\int_0^{\Delta^+}u^+(R^+-y^+)\mathrm{d}y^+\right] \times\left[R^+-\int_0^{\Delta^+}u^+(R^+-y^+)\mathrm{d}y^+\right]\Big/ R^+(R^+-\Delta^+)\left(\frac{1}{Sc}+\frac{\varepsilon_D}{\nu}\right)\right\}\mathrm{d}\Delta^+. \quad (21)$$

From the definitions of the local Sherwood number and mass transfer Stanton number, Sh_x and St_x become

$$Sh_x=\frac{ScRe\sqrt{f/2}}{(w_w^+-w_b^+)} \quad (22)$$

$$St_x=\frac{\sqrt{f/2}}{(w_w^+-w_b^+)} \quad (23)$$

where w_b^+ is the bulk mass fraction which can be shown from the mass balance equation to be

$$w_b^+=\frac{4x^+}{Re} \quad (24)$$

and f is the friction coefficient corresponding to the maximum drag reduction asymptote as proposed by Virk, Mickley and Smith [5] as follows:

$$\frac{1}{\sqrt{f}}=19.0\log_{10}(Re\sqrt{f})-32.4. \quad (25)$$

RESULTS AND DISCUSSION

In the calculation of the above parameters, numerical integration formulae introduced by Minkowycz and Sparrow [11] were used. These integration formulas were derived by fitting a third-degree polynomial through four points. The convergence of each numerical integration was also ensured by increasing the number of panels from 50 to 150.

MASS TRANSFER ENTRANCE LENGTH

The mass transfer entrance length for drag-reducing viscoelastic fluids calculated using equation (21) is shown in Fig. 1. For Schmidt number equal to 1000, which applies to many drag-reducing polymer solutions, the mass transfer entrance length becomes 8–40 diameters depending on the Reynolds number. Considering the extremely long heat transfer entrance length ($L/d \sim 400$) reported by Yoo and Hartnett [12] and Ng *et al.* [13, 14], the short mass transfer entry length is somewhat surprising if we assume that heat and mass transfer are analogous processes.

SHERWOOD NUMBER

The Sherwood number for fully established mass transfer conditions was calculated as a function of Reynolds number for three different Schmidt numbers using equations (18), (20) and (24) together with $\Delta^+ =$

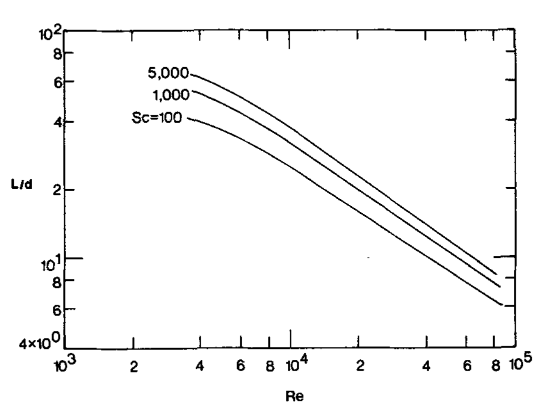

FIG. 1. Mass transfer entry length vs Reynolds number for drag-reducing viscoelastic fluids.

R^+. The results are presented in Fig. 2, which also shows the Newtonian results for Notter and Sleicher [2] for Sc equal to 1000. Compared to the Newtonian values, the current results for a viscoelastic fluid with Sc equal to 1000 give 65–75% mass transfer reduction in Sherwood number depending on the Reynolds number.

THE MASS TRANSFER j FACTOR

In the correlation of mass (or heat) transfer data for Newtonian fluids, the j factor has been widely used since it has been empirically shown that it absorbs the Schmidt (or Prandtl) number effect and consequently the j factor is a function only of the Reynolds number. For purely viscous non-Newtonian fluids, Bird [15] demonstrated in his modification of Graetz solution that j_H eliminates the Prandtl number effect. Therefore, it has been of interest to test the applicability of the above statement to viscoelastic fluids.

The j factor of mass transfer calculated as a function of the Reynolds number is shown in Fig. 3. The current calculation gives an identical value of j_D for three different Schmidt numbers at each Reynolds number, suggesting that the j factor expression effectively eliminates the Schmidt number effect in the correlation of mass transfer data. The current results shown in Fig. 3 can be described by the following correlation

$$j_D = 0.022Re^{-0.28}. \tag{26}$$

COMPARISON WITH EXPERIMENTAL DATA

There are numerous papers reporting turbulent mass transfer rates for turbulent flow of Newtonian fluids in channels of various shapes [16–24]. In such cases there is conclusive evidence that an analogy between momentum, heat and mass transfer can be drawn as follows [25, 26]:

$$j_H = j_D = f/2. \tag{27}$$

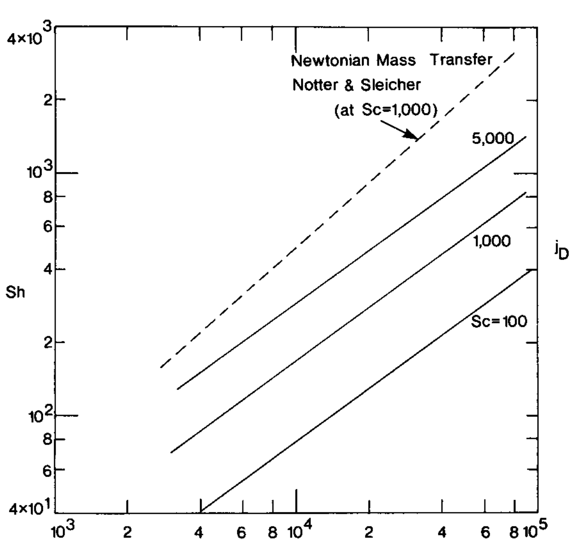

FIG. 2. Sherwood number vs Reynolds number for viscoelastic fluids.

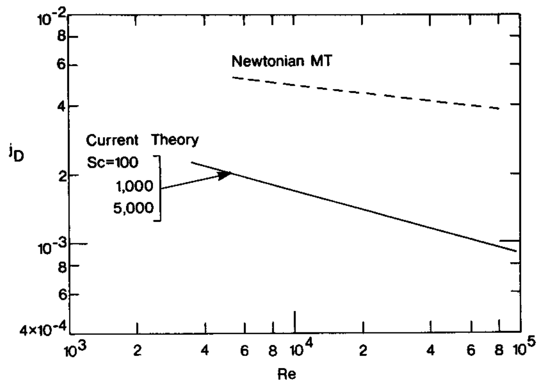

FIG. 3. Mass transfer j factor vs Reynolds number for viscoelastic fluids.

Turning to viscoelastic fluids, relatively few experimental mass transfer data exist [27–30]. For these studies the electrochemical method is the experimental technique most commonly used to measure the mass transfer rate at the wall (an excellent and detailed review of the electrochemical method was reported by Mizushina [31]).

In contrast, Virk and Suraiya [30] recently applied both the weight loss method [18] and the ultraviolet spectrophotometric method. Unlike the previous investigators [27–29] who used relatively short mass transfer test sections of l/d less than four, Virk and Suraiya constructed mass transfer sections of 34.5 and 69 diameter lengths preceded by a hydrodynamic section of 129 diameter lengths and conducted turbulent mass transfer measurements with aqueous solutions of polyethylene oxide. The mass transfer rates measured in the 69 diameter length test section gave identical values to those obtained in the 34.5 diameter length tube for all of the polymer solutions used in the Reynolds number ranging from 5000 to 35 000. Using those data obtained in the shorter test section, they proposed the following expression for the maximum mass transfer reduction asymptote

$$j_D = 0.022Re^{-0.29}. \tag{28}$$

This correlation equation is shown in Fig. 4.

Shulman and Pokryvailo [10] conducted turbulent mass transfer measurements with dilute aqueous solutions of polyethylene oxide in three different sizes of square channel (1 cm × 1 cm, 2 cm × 2 cm and 5 cm × 5 cm) using the electrochemical method and proposed a similar correlation for the mass transfer reduction asymptote

$$j_D = 0.0206Re^{-0.275}. \tag{29}$$

This is also shown in Fig. 4. The current predictions using successive approximation show good agreement with the empirical correlations proposed by Virk and Suraiya [30] and Shulman and Pokryvailo [10], lending support to the accuracy of the numerical scheme for analyzing turbulent mass transfer in channel flows.

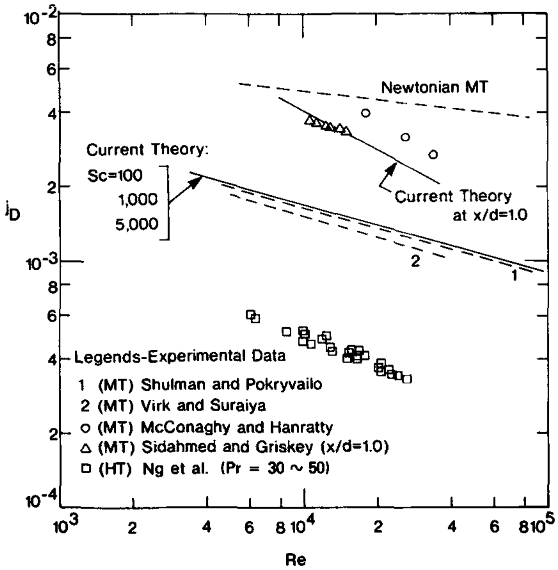

FIG. 4. Comparison with empirical mass transfer data, mass transfer j factor vs Reynolds number.

The pioneering work of Sidahmed and Griskey [27] is also presented in Fig. 4. These results were obtained in a one diameter length mass transfer test section with aqueous solutions of polyethylene oxide. Our present prediction at x/d equal to 1.0 gives excellent agreement with the experimental results of Sidahmed and Griskey. In contrast, data reported by McConaghy and Hanratty [28] who used a test section of approximately three diameter length and an aqueous solution of polyacrylamide (Separan AP-30), are substantially higher than our current predictions even at x/d equal to 1.0. This may be attributed to the severe chemical degradation of Separan solutions [32] in the presence of an electrolyte.

Figure 4 also shows the heat transfer results for viscoelastic fluids by Ng, Cho and Hartnett [14] which were obtained in a test tube of l/d equal to 430 with moderately concentrated aqueous solutions of polyethylene oxide and polyacrylamide. The comparison of the mass transfer results with heat transfer data of Ng, Cho and Hartnett clearly demonstrates that the maximum heat transfer reduction asymptote is smaller than the maximum mass transfer reduction asymptote by a factor of three. The present results indicate that the eddy diffusivity of mass is much greater than that of heat (see Usui [33] and Mizushina and Usui [34]) and therefore, the analogy between heat and mass transfer does not apply to viscoelastic fluids. This observation is consistent with our previous arguments concerning entry lengths of heat and mass transfer for viscoelastic fluids. The above comparisons of heat and mass transfer in undegraded saturated viscoelastic fluids may be summarized in the following inequalities

$$\varepsilon_H < \varepsilon_D \qquad (30)$$

$$j_H < j_D \qquad (31)$$

$$L_H > L_D. \qquad (32)$$

CONCLUSIONS

The successive approximation is a simple and effective numerical scheme for analyzing the turbulent mass (or heat) transfer phenomenon in channel flows. The current analytical study rests on the validity of the eddy diffusivity of mass for polymer solutions proposed by Shulman and Pokryvailo [10] and Virk's velocity profile corresponding to maximum mass transfer reduction asymptote. Under the acceptance of these two conditions, the following conclusions can be drawn:

(1) The predicted maximum mass transfer reduction asymptote for viscoelastic fluids which is in excellent agreement with empirical results [10, 27, 30] reported in the literature may be described by the following correlation:

$$j_D = 0.022Re^{-0.28}.$$

(2) The mass transfer entry length for drag-reducing fluids in circular pipe flow is found to be 8 to 40 pipe diameters depending on the Reynolds number.

(3) There is no simple and direct analogy between heat and mass transfer for drag-reducing viscoelastic fluids.

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TRANSFERT MASSIQUE DANS UN ECOULEMENT EN CONDUITE DE FLUIDES VISCOELASTIQUES

Résumé—On calcule analytiquement la longueur d'entrée du transfert massique et l'asymptote de réduction maximale de transfert massique pour la réduction de frottement des fluides viscoélastiques. La diffusivité turbulente massique de Shulman *et al.* [10] et le profil de vitesse de Virk correspondant à la réduction maximale de transfert massique dans l'écoulement turbulent en conduite sont appliqués à cette étude. La longueur d'entrée pour le transfert massique est entre 8 et 40 diamètres du tube selon le nombre de Reynolds. Les flux massiques calculés sont en bon accord avec les résultats empiriques disponibles et montrent une réduction de 65 à 75% environ du flux massique en comparaison avec les valeurs correspondant au fluide newtonien pour les mêmes nombres de Reynolds et de Schmidt. Une comparaison de ces résultats avec des mesures de transfert thermique disponibles permet de conclure qu'il n'y a pas de relation directe simple entre les transferts de chaleur et de masse pour les fluides viscoélastiques à réduction de frottement.

STOFFTRANSPORT BEI TURBULENTER ROHRSTRÖMUNG VON VISKO-ELASTISCHEN FLUIDEN

Zusammenfassung — Die Anlauflänge der Stoffübertragung und die Asymptote der maximalen Stoffübergangsreduktion für widerstandsmindernde visko-elastische Fluide wurden analytisch nach dem Verfahren der schrittweisen Approximation berechnet. Der turbulente Diffusionskoeffizient nach Shulman *et al.* [10] sowie das Geschwindigkeitsprofil nach Virk, das der maximalen Stoffübergangsreduktion bei turbulenter Rohrströmung entspricht, wurden in der vorliegenden Untersuchung verwendet. Für die Anlaufstrecke des Stoffübergangs dieser Fluide wurden je nach der Reynolds-Zahl Längen von 8 bis 40 Rohrdurchmessern gefunden. Die berechneten Stoffübergangszahlen dieser Fluide stimmen gut mit den vorhandenen empirischen Daten überein. Sie zeigen näherungsweise eine Abnahme von 65 $\div$ 75% der Stoffübergangswerte gegenüber den Werten Newtonscher Flüssigkeiten bei gleichen Reynolds- und Schmidt-Zahlen. Ein Vergleich dieser Stoffübergangsergebnisse mit vorhandenen Ergebnissen von Wärmeübergangsmessungen führt zu dem Schluß, daß es keinen einfachen, direkten Zusammenhang zwischen Wärme- und Stoffübergang bei widerstandsmindernden visko-elastischen Fluiden gibt.

МАССОПЕРЕНОС В ТУРБУЛЕНТНОМ ПОТОКЕ ВЯЗКОПЛАСТИЧНЫХ ЖИДКОСТЕЙ В ТРУБЕ

Аннотация — Длина входного участка и величина максимального снижения массопереноса для снижающих сопротивление вязкопластичных жидкостей были рассчитаны аналитически с использованием метода последовательных приближений. В данном анализе были применены вихревая диффузия, представленная в работе [10] и профиль скорости Вирка, соответствующий максимальному снижению массопереноса в турбулентном потоке в трубе. Было найдено, что длина входного участка равна 8 ÷ 40 диаметров трубы в зависимости от числа Рейнольдса. Вычисленные скорости массопереноса этих жидкостей хорошо согласуются с имеющимися эмпирическими данными по массопереносу, демонстрирующими примерно 65–75% снижение скорости массопереноса по сравнению с ньютоновскими жидкостями для тех же чисел Рейнольдса и Шмидта. Сравнение данных результатов по массопереносу с имеющимися результатами измерений теплопереноса позволило сделать вывод о том, что не существует простой корреляции между тепло- и массопереносом снижающих сопротивление вязкопластичных жидкостей.